Using constraints at the conceptual stage of the design of carton erection

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Abstract

Cartons are a common way to package many consumer goods. If new designs are used it is necessary to simulate their erection to ensure that this works correctly, particularly at high speed. In this paper, constraint-based techniques are used to model the carton itself and to provide such a simulation. Optimisation is used to resolve constraints and this can also be used to improve the erection process. It is also possible to model the mechanisms used to erect cartons within the same constraint modelling environment and hence simulate and improve the way in which carton faces are driven and guided.

Keywords: packaging, cartons, carton erection, constraints, geometric constraints

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1. Introduction

There are two main forms of material in use for packaging such commodities as food items and electronic goods. These are plastic film and carton-board [1, 2]. Today, the latter is often seen as a better option from an environmental point of view as it is a "natural" material and hence is more environmentally friendly [3]. However, it has a reputation for being used only for very basic packaging shapes. There is interest in different forms of geometry in order to aid marketing [4, 5], and also to package variant forms of product [6].

Cartons are normally supplied to a packer as flat nets. They need to be "erected" into their final shape. This can be done manually or, preferably, by mechanical means [3]. New shapes potentially mean new designs of dedicated machines or the use of reconfigurable equipment [7]. There is then a need to model the erection process to ensure that it works as expected (at high speed) and, in particular, that there is no unwanted interference between parts of the net as it is folded. Such a model can be achieved by setting up explicit equations and then solving these [8]. However this offers little flexibility. What is more useful is some form of visual simulation, (such as that used for training [9] and for showing the kinematics of pop-up books [10]). Some relevant simulations have been created (e.g. [11, 12, 13, 14, 15]). However these are based on finite element techniques and hence specialised expertise is required to create and use them.

This paper looks at the use of constraint-based techniques in simulating and improving the erection process. There are three aims.

- The first is to show that constraints can indeed be used to describe the relations inherent in the geometry of a carton net and hence create a (simple) simulation.
- The second is to illustrate that the constraint-based description can be used as starting point for investigating improvements in how the erection is carried out.
- The third is to show that the constraint-based model can be extended to include details of the mechanism(s) used to drive and control the process.

The first aim is considered with respect to a general carton net. The second and third aims always relate to specific details of a particular design. For this reason, the approach is considered in application to a particular carton form. The approach is intended as an aid in the initial stages of a design. It makes use of interaction between a constraint-based software environment and a human user so that alternatives that occur to the designer can be investigated easily. Once the approach has led to a potential design solution, more sophisticated mathematical and/or computer modelling can be undertaken, for example to check that unwanted clashing between carton and machine does not occur.

The approach proposed forms an example of the use of geometric constraints [25, 26, 27]. The basic carton net can be described in terms of the positions of faces and the transforms between them. When loops of faces exist, some of these move as a result of others being explicitly driven. Constraints can be applied to model this situation and to determine the angles of the following faces. This is discussed in section 2.

Section 3 illustrates a constraint-based simulation using a particular sample carton. It is this carton that is used as the illustrative example in later sections. The system used to resolve constraints is based on optimisation. This means that one can consider trying to improve performance parameters. An example is discussed in which the opening available for inserting product is maximised.

The required guiding mechanisms can also be modelled using the constraint-based approach within the same environment as the carton. This is described in section 4 and examples of finger mechanisms to guide the erection of the sample carton are discussed.

2. Background

In the area of mechanism and machine design a great deal of analysis has been undertaken. However much of this depends upon the form of the mechanism being known and possibly also requires specialised knowledge (e.g. [16, 17]).

In practice, a designer of packaging machinery is presented with a proposed new form of carton and is expected to create an appropriate means for erecting it automatically. Here one is working at the conceptual stage of the design process. This requires exploration of what is possible and the creation of ideas for discussion and further refinement [18, 19]. This may be aided by catalogues of potential design concepts or previously successful approaches [20, 21, 22]. A full mathematical model is impossible until the design has been further defined.

What is required is an approach which allows a designer to capture what initial information is available about the design, and this certainly includes the geometric definition of the carton net. The approach should allow a simulation of an erection process to be created simply. If it also allows interaction with a user, then the simulation can be the basis for investigating different erection strategies and selecting and optimising the most appropriate.

One approach to conceptual design is based on the idea of constraints [23]. In the initial stages, constraints are often more evident as these bound what can be done. As a design evolves, more and more constraints become evident and the understanding of the design task increases. Constraints can be used in three ways [24]: for constraint monitoring in which the known constraints are checked for any violations; for constraint satisfaction in which an automatic search is made for design parameters which allow all the constraints to be satisfied; and constraint optimisation in which again a feasible design is sought with the requirement also to optimise one or more performance measures.

3. Use of constraints

Typically a carton is erected by folding it from an initially flat blank. The blank as supplied has already been cut and its creases have been formed by pressing thin metal rules into it [1, 3]. The erection process mainly requires the formation of folds around these creases.

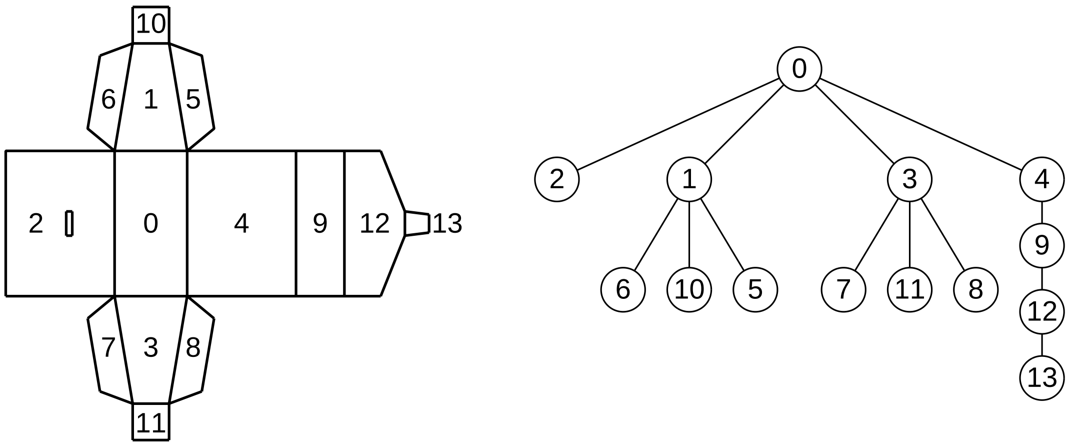

Figure 1: Loop-free carton net and face graph based on example in [8]

Figure 1 shows a typical form of carton net (based on an example in [8]). Also shown is its face graph [28, 29]. This has a node corresponding to each face of the net and two nodes are joined if their faces are adjacent. For this example the graph is a tree, there are no loops. This means that each of the creases can be folded independently of the others. In practice, such folding is carried out by various mechanisms (possibly working together as part of a large machine).

A simulation of the erection process can be achieved by creating computer models of each face and applying rotary transforms so that each face turns about the crease which connects it to its neighbour closer to the root node, labelled 0 [30]. The angles of rotation can be given and take values running from zero to the (application dependent) final value. The independence of the values means there are many choices for these sequences of angles, although some limitation exists because of the need to avoid clashing between faces.

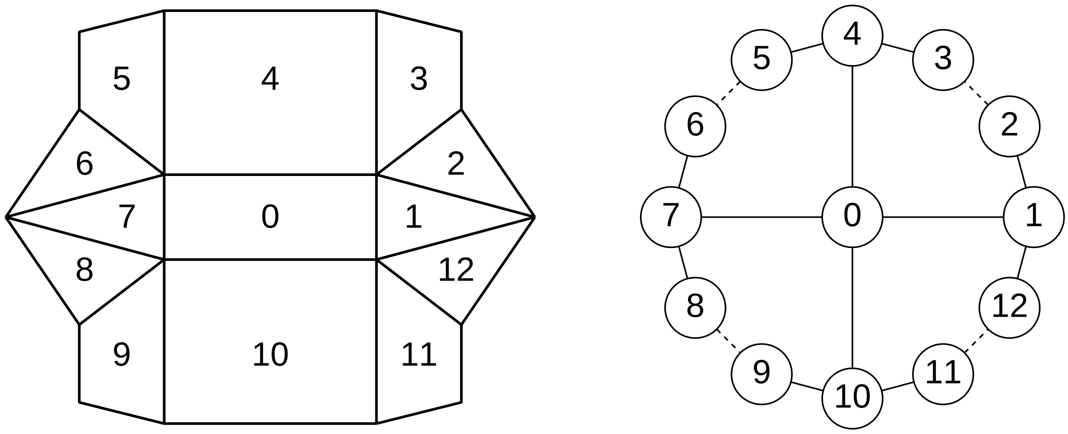

Figure 2: Face graph of carton net

Consider the variant carton design shown in figure 2 (based on examples in [4, 28, 31]). Its face graph now has loops. These correspond to what are here referred to as "gusset" corners. An example is the corner associated with faces 0, 1, 2, 3, 4. The carton can be erected by holding the base face 0 fixed and rotating faces 1 and 4. This has the effect of requiring faces 2 and 3 to move. There is no explicit need to drive these with external mechanisms, although this may be necessary to ensure that the two gusset faces move in the correct direction.

The graph of the carton can be modified by removing edges until a spanning tree remains. There are several ways in which this can be done and one is suggested by the dotted edges in the graph on the right of figure 2. These are the edges corresponding to adjacency of gussets faces. The edges of the spanning tree represent those creases whose folding needs to be driven. The other creases follow but possibly need guidance.

In simulating the erection process in the early stages of the design process, the angles for creases represented in the spanning tree can be provided conveniently via a data table; they may need to be modified as the design process proceeds and a greater understanding is obtained by the designer. Each set of values determines the angles of the gusset faces and these need to be calculated. This can be done analytically by considering the equations needed to ensure that faces come together correctly. However it is easier to use computational means and, in particular, the ideas of geometric constraints [25]. Figure 3 shows one of the corners for the carton in figure 2. Two of the main side faces, one rectangular and the other triangular (shown shaded), have been rotated. The gusset faces have moved with them. The basic constraint is to bring them together; this is equivalent to making the two corners A and B in the figure coincide. In mechanism terms, the two faces form a simple dyad.

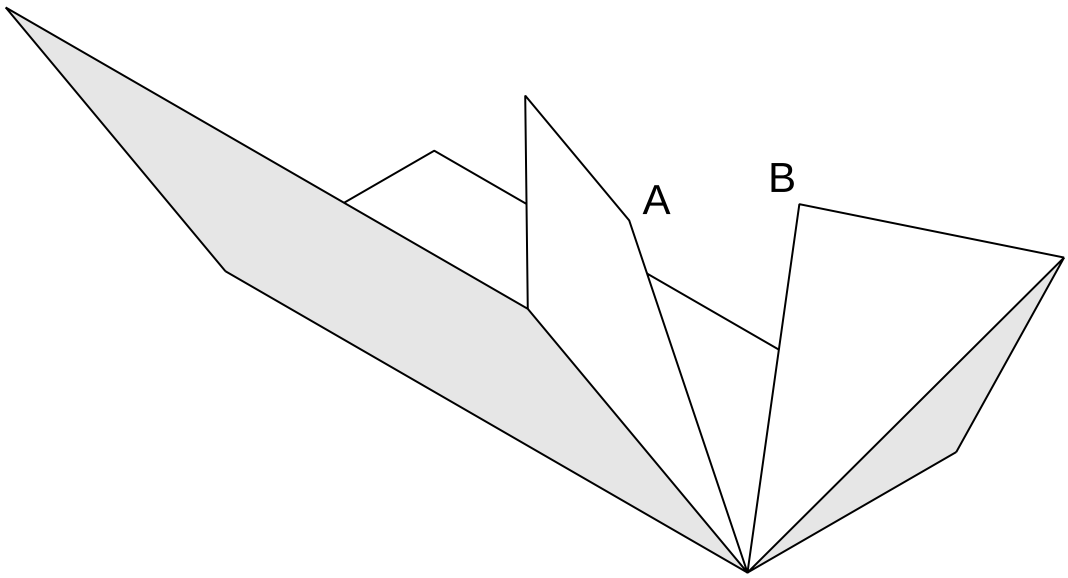

Figure 3: Carton faces forming a dyad

There are a number of geometric constraint solving approaches available (e.g. [26, 27]). The one used for the examples in this paper is a constraint modelling environment [32] which allows the creation of wire-frame graphical entities including points, lines and faces (which can be shaded). These are defined via a user interface language which also allows constraints to be defined between entities. Also specified are the parameters which are allowed to vary when the constraints are resolved. The graphical user interface for the constraint modelling environment is shown in figure 4.

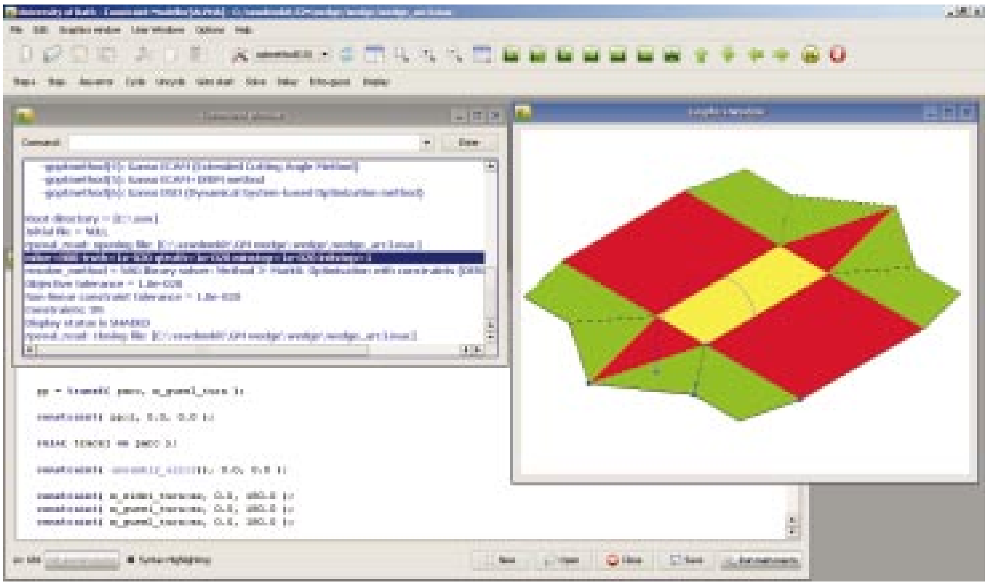

Figure 4: User interface for constraint modelling environment showing the carton erection model

Constraint resolution is carried out using optimisation techniques [26, 33]. Each geometric constraint effectively represents a distance between entities and when entities are together such distances are zero. In the case of the dyad shown in figure 3, the constraint is specified by a command of the form

```
rule( A on B );
```

Here `on` is an in-built binary function which finds the distance between entities. If points A and B have have positions (A_x, A_y, A_z) and (B_x, B_y, B_z) in the global space, then the expression `A on B` is evaluated as the following

$$\sqrt{[}(A_x - B_x)^2 + (A_y - B_y)^2 + (A_z - B_z)^2]$$

which, of course, is zero when the points coincide. The points are actually defined with respect to local coordinate systems within the two faces in which they lie. To find their positions in global space, the system applies rigid body transforms as dictated by the hierarchy of the spanning tree [30, 32]. In particular, in satisfying the above constraint, the system adjusts the angles of the two gusset panels in which the points lie.

When more than one constraint is imposed, the environment forms the sum of the squares of the constraint values and treats this as a function of the variable parameters.

As the form of the imposed constraint is not known a priori, direct search optimisation techniques [34, 35] are the appropriate for constraint resolution. The constraint modelling environment has access to a number of standard methods including: Hooke and Jeeves [34], Powell's direct search method [34], the quasi-Newton method (`fminunc()`) of the MATLAB optimization toolbox [36, 34], the sequential quadratic programming (SQP) method from the NAG library [37], and the derivative-free bundle method (DFBM) of the

GANSO (global and non-smooth optimization) library [38, 39]. The method used for all the examples in this paper is Powell's direct search method.

The following is sample code for the constraint modelling environment which provides the constraints for the gusset corner in figure 3.

```
function Resolve
{
    var FaceA_rotation, FaceB_rotation;


    rule( A on B );


    rule( FaceA_rotation >= 0 );
    rule( FaceA_rotation <= 180 );
    rule( FaceB_rotation >= 0 );
    rule( FaceB_rotation <= 180 );
}
```

The transforms for the two faces are set up so that they represent rotations about the edge joining each to its neighbour. The `var` statement allows the rotations angles to be changed and the final four `rule` commands ensure that folding takes place in the correct direction.

When the function is invoked, the constraint modelling environment automatically resolves the constraints. Similar functions are used to deal with the other corners.

There are two advantages in using optimisation techniques to resolve the imposed constraints. Firstly it allows progress to be made even when the constraints are in conflict. The system can still obtain a solution which is some form of "best compromise". This is helpful in the early design stages when understanding of the task is still limited. Secondly, it allows ways for improvement to be investigated. If a desired performance measure is specified as part of a constraint, then the system can be used to adjust parameters to try to optimise this measure. It is this facility which is discussed in later sections.

4. Carton simulation

In this section it is shown how the constraint-based description of a carton net can be used to create a simulation of its erection. The second carton discussed in the previous section is used as an example although the approach can be used with any net. Its dimensions are given in figure 5.

As previously discussed, constraints can be set up to determine the posi-

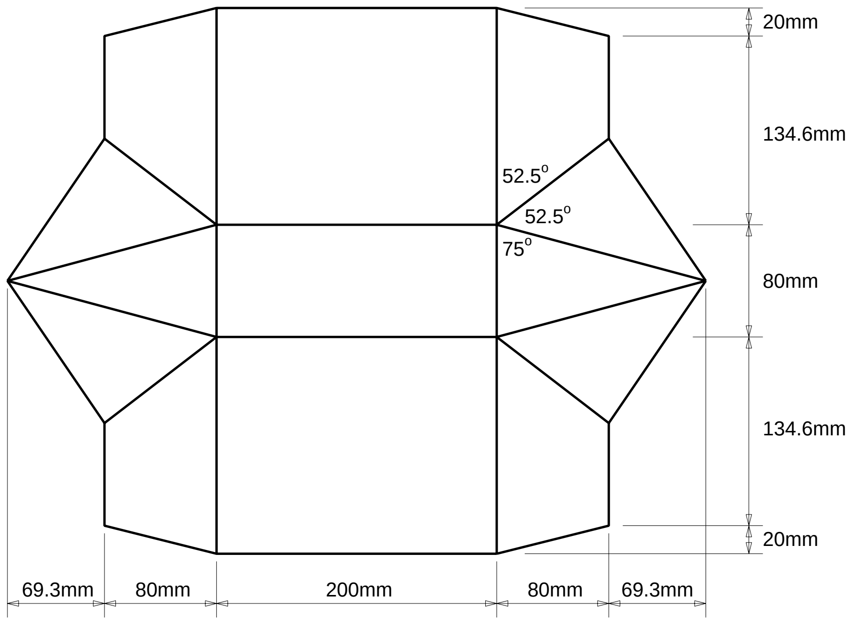

Figure 5: Initial net for carton

tions of the gusset faces while the main faces are driven to their end positions. Stages in the resultant simulation are given in figure 6.

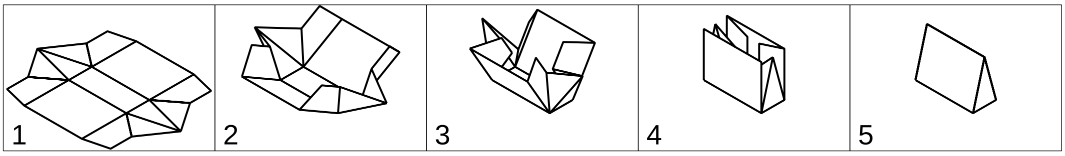

Figure 6: Stages in carton erection

While the equations involved in the performing the simulation are simple enough to be solved analytically, there are advantages in dealing with them computationally. One of these is the ability to use the simulation for visual checking of the erection process. At the early stages of a design, such visual checking is adequate and straightforward to apply. More sophisticated checking for interpenetration of faces can be undertaken later in the design process.

It is also possible to identify interpenetration using the constraint modelling environment. To do this, the carton faces are additionally represented as solid objects of small thickness. The transforms used to position the original faces are also applied to the solids so that they move correctly during the simulation of the erection process. Part (a) of figure 7 shows these solid faces, and part (b) shows them again with exaggerated thickness for clarity. As the simulation progresses, the system evaluates the volumes of the intersections of pairs of face solids. When a volume becomes non-zero, interpenetration is occurring. In this way constraint monitoring can be achieved [40]. Part (c) of figure 7 shows the four solid intersections corresponding to the interpenetration seen in part (a) of the figure.

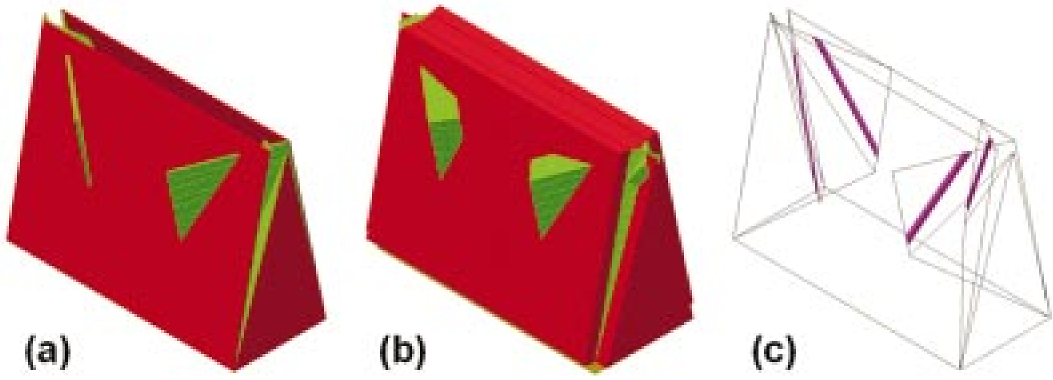

Figure 7: Use of solid intersection to identify interference of carton faces

The simulation and indeed the physical erection process are governed by the closing of the four main faces, with the gussets corners following. The two triangular faces turn through a right angle and the two rectangular faces turn through 105 degrees. The simplest way to achieve these is to create the data table of driving angles so that each pair of these driving faces steps through the same number of equally spaced steps. However when this is attempted, for this particular carton, interference between faces occurs. Examples can be seen in figure 8 (with the case on the right corresponding to the wire-frame version of part (a) of figure 7).

In the early part of the motion, opposite pairs of gusset faces intersect, and when the erection is almost complete, the gusset faces penetrate the rectangular sides.

The constraint-based approach has been used to detect a problem with the user's selection of driving angles. Once this problem is realised, it can be avoided by a better choice of those angles. In the simulation in figure 6, the designer has chosen to take all four driving faces firstly in equal steps through 90 degrees. Finally, the rectangular sides are turned through the last 15 degrees. The simulation shows that now no clashing occurs.

Constraints could have been introduced to allow the software to find appropriate closing strategies to avoid the interference. But it is clearly easier to take advantage of the human user's abilities. However, the need for clash avoidance has required that the rectangular and triangular side panels can be controlled and driven independently. So it is worth investigating whether advantage of this can be taken by investigating alternative closing strategies. This is used here simply as illustration of how additional constraints can be added to the model to consider improvements in the erection process. Such

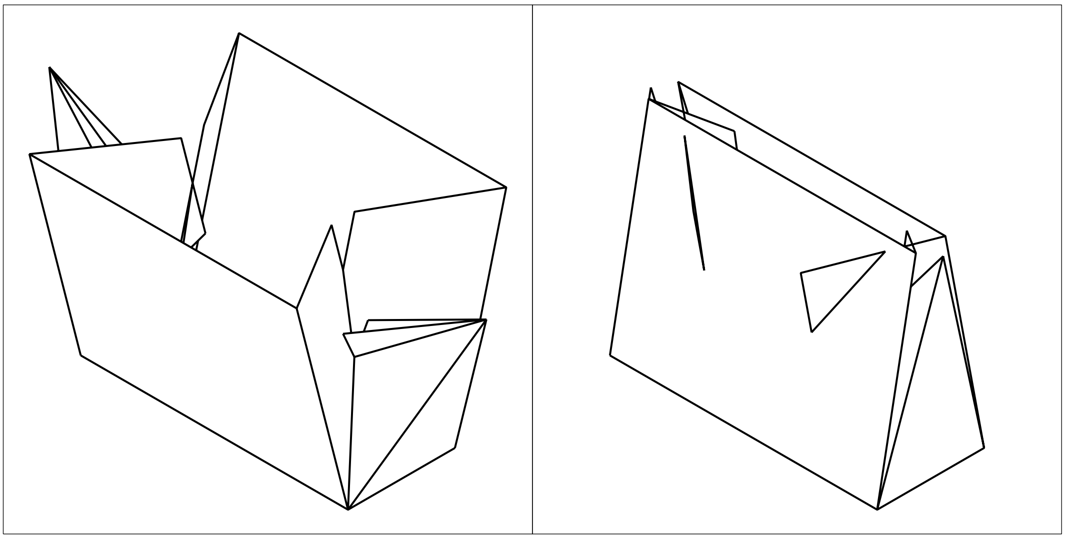

Figure 8: Interference between carton faces

improvements depend upon the form of the carton and considerations about its use.

The need to fill the carton with product can be considered. Product needs to be dropped into the carton at some point and this could possibly be done without stopping the erection process. Filling must occur after the point when the angles main panels ensure that the product can be retained, and before the point when the opening at the top of the carton does not permit product to enter. This of course is dependent upon the product itself, the rating of filling, and the filling head that is used.

On the left of figure 9 are an oblique and plan view of the carton from above when the rectangular faces have been turned through 60 degrees. This is for the closing regime in which the rectangular and triangular faces turn initially through the same steps. While there is a large opening for product to enter, the rotation of the gusset may not ensure that product cannot escape.

What is considered here is the problem of keeping the opening at the top of the carton maximal while erection takes place. To investigate this, an additional geometric constraint is added to the simulation model. This relates to points at the top of the triangular faces and at the top of the crease between the gusset faces. Figure 9 shows points P and Q which are examples of these two sets of points respectively. The horizontal distances of these points from the centre of the base of the carton are found. The additional constraint is imposed to maximise the smaller of these distances. If (P_x, P_y, P_z) and (Q_x, Q_y, Q_z) are the positions of the points in global space then the constraint rule is set up to maximise the following expression; the origin of the global coordinates is at the centre of the base panel.

$$\max\left(\surd[P_x^2 + P_y^2],\ \surd[Q_x^2 + Q_y^2] \right)$$

To resolve the constraint for any given angles of the rectangular faces, the simulation is allowed to vary the angle of the triangular faces. Both rectangular faces are assumed to have the same angle, as are the two triangular ones. The effect of the constraint is also to make the two distances both equal to the optimal value.

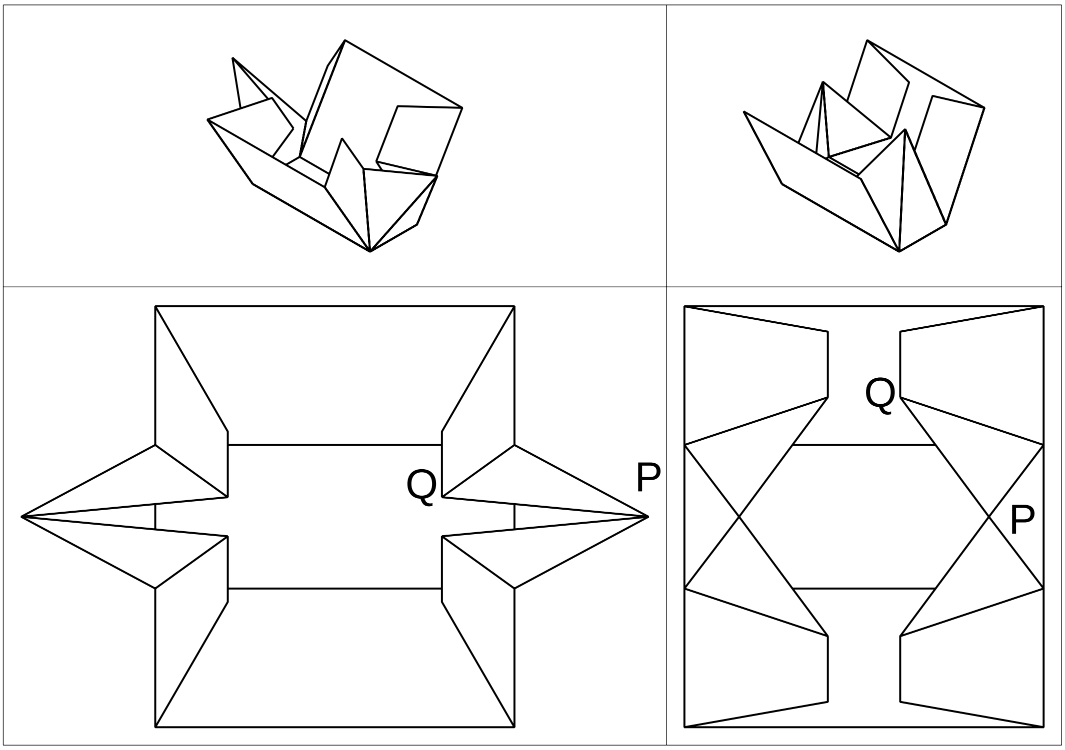

Figure 9: Area of partially closed carton available for fill under different erection regimes

It is found, from the simulation, that the additional constraint can be resolved successfully when the angle of the rectangular face lies between 24 and 64 degrees. The corresponding values for the triangular faces are found to be 96 and 102 degrees. Note that these are close together and both represent an over-folding of the triangular faces. In this range a configuration such as that on the right of figure 9 exists. For angles smaller than 24 degrees, only the equivalent of the configuration on the left of the figure can be found. The motion in the initial stages can be obtained by driving the rectangular and triangular faces in equal angular steps up to 24 and 96 degrees respectively. For angles larger than 64 degrees, some of the gusset faces start to penetrate the rectangular face. To deal with this, the additional constraint is replaced by one which specifies the top of the gusset crease, point Q, must lie on the rectangular face. This ensures that penetration cannot occur.

Adding each of the extra constraints establishes a relation between the angles of the two sets of driving faces. This is shown on the left in figure 10. In the initial straight portion, the angles are independent but are chosen to increment linearly. In the second portion of the motion, the angles are related so that the opening is maximised. In the final part, the angles are again related, this time to avoid penetration of the rectangular faces. To achieve such motions in practice, the triangular and rectangular faces need to be driven by separate controllers. Some smoothing between the three different phases of the motion is necessary to avoid adverse jerk and this is indicated in the graph on the right of figure 10.

In this section constraints have been used to provide a simulation of the

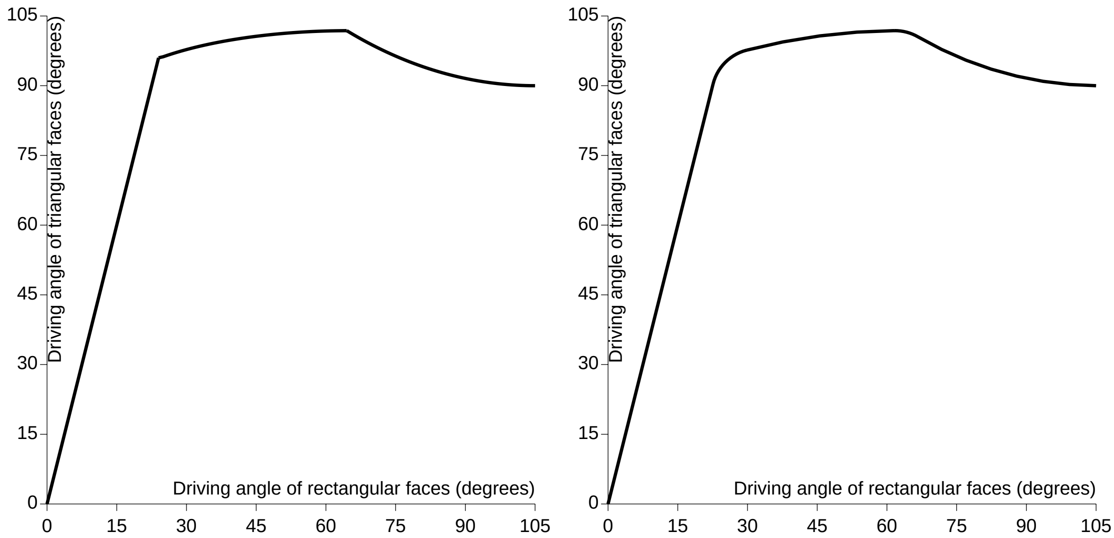

Figure 10: Relation between driving angles for rectangular and triangular faces for improved central area of carton filling: left - initial relation; right - with smoothing to avoid induced jerk

erection process and hence detect a problem with regime used to fold the main faces. The introduction of additional constraints has allowed the designer to investigate (and avoid) problems of interpenetration and to improve the erection process to meet the needs of the application.

5. Guidance

This section is concerned with the use of additional constraints to investigate how mechanisms can be introduced to drive and control the motion of the carton panels during folding. This is illustrated by reference to the example carton used previously and the needs to guide its gusset flaps to ensure their correct motion.

In carton erection, some faces need to be actively pushed so that they fold about their creases. These are the creases corresponding to edges in the spanning tree of the face graph. Other creases are then induced to fold. These are the ones between gusset faces. As normally there are two ways in which the dyad of faces can be configured, some guidance is required to ensure that these creases turn as required. In practice, this may only need to be a push in the right direction for the first few degrees of rotation of the neighbouring faces. However it may be necessary to provide the guiding motion for a longer period. As in [8], this motion is investigated over the complete erection process. Once the motion is determined, it is then possible to obtain the parameters needed to control the guiding mechanisms.

The carton considered previously (figure 5) is again used as an example. The challenge is to find a guiding mechanism which initially lies below the plane of the net so that it can push the gusset faces upwards, and which finally follows the folded gusset into the closing carton. The simplest mechanism is a thin finger which rotates in an arc. So that the finger can lie inside the final carton without distortion, the arc needs to lie in the plane of the final position of the relevant rectangular side.

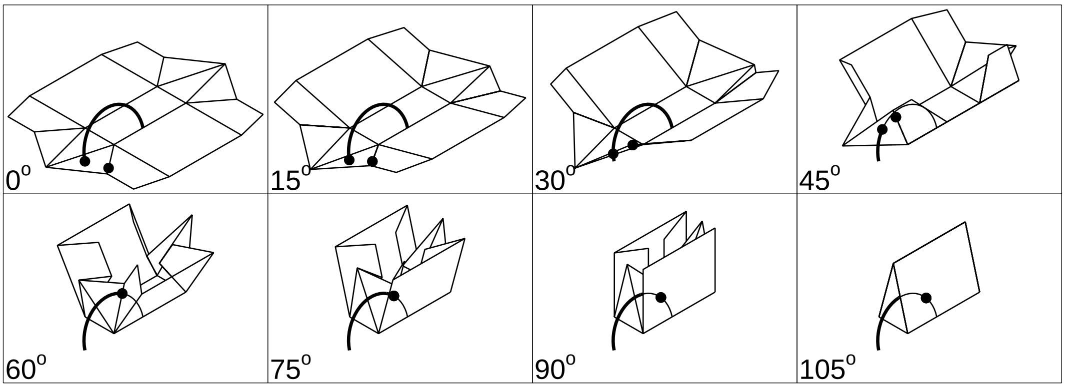

Figure 11: Simulation of erection when using guidance with one degree of freedom

Figure 11 shows stages in the simulation of a motion based on this idea. The arc is shown as a semicircle. Geometric constraints are applied as before to define the carton during its erection. Two additional constraints are present. The first is that a point (shown as one of the dots in figure 11) must lie on the arc and also on the gusset face adjacent to the main triangular face. This represents the point where the guiding finger makes contact with the carton. The other dot in the figure is a point on the crease between the two gusset faces. The second constraint is that the two points should be coincident. This represents the desire that the finger should push along that crease.

In the early part of the motion, the second constraint cannot be satisfied. However, as optimisation is used to resolve the constraints, the simulation attempts to minimise the distance between the two points. This means that the finger is encouraged to move towards the crease and so have a greater pushing effect. For this to happen, there is a tendency for the triangular side to stay low and, for part of the process, to turn below its original position.

It is found from the simulation that when the rectangular sides have turned through 52.5 degrees, the two points can indeed coincide. The finger has now reached the crease and it stays in contact with it for the rest of the motion. As before, applying the additional constraints establishes a relation between the driving angles required for the rectangular and triangular faces.

This relation is shown in figure 12. The attraction of this mechanism is its simplicity with just one degree of freedom to control. However, as noted above, it is found from the simulation that the triangular face turns downwards until the rectangular face has reached 30 degrees This means that this guidance solution may not be acceptable in practice.

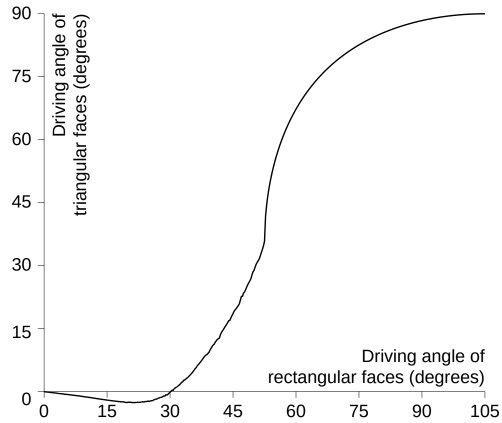

Figure 12: Relation between driving angles when using guidance with one degree of freedom

An alternative approach with more degrees of freedom is now discussed. It comprises a small PRRR manipulator with three rotary links and whose base is allowed to slide along a fixed track. The manipulator is shown in figure 13 together with its position relative to the initial flat net. This is similar to a technique used in [8] where the required trajectories are determined by explicit calculation. Here the simulation is created via the constraint-based environment and this is used to determine the trajectories and hence the control requirements.

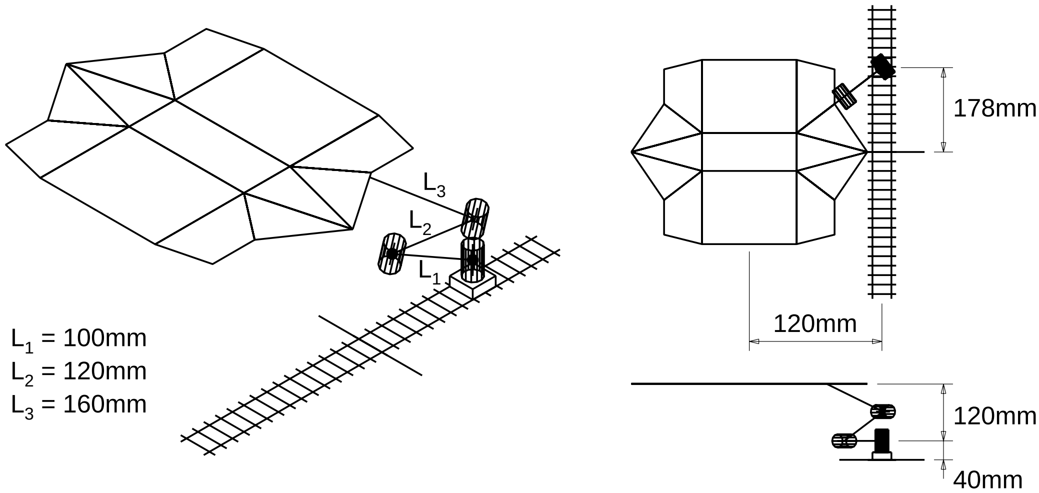

Figure 13: Manipulator and initial carton

The manipulator is described within the software environment as a “stick model”. Each link is related to its neighbour nearer to the world space by a transform matrix [30] producing a hierarchy similar to that for the carton itself (cf. figure 1). When constraints involving the manipulator are resolved, there are four degrees of freedom: the angles of the three links and the linear position of the base.

The closing strategy used for the carton is that which optimises the central area as discussed in section 3. The relation between the angles for the rectangular and triangular sides is given in figure 10. The graph on the right of the figure is used; this is the obtained from the graph of the left by smoothing the corners so as to ensure a better motion.

The defined motion of the carton is now used to derive the motion of the manipulator. The following pseudocode shows the function `Manipulatorsolve` which deals with the interaction between the manipulator and the carton.

```
function Manipulatorsolve
{
    var Arm1_rotation;
    var Arm2_rotation;
    var Arm3_rotation;
    var Base_translation;

    Redefine_midplane();
    rule( Push1 on Parm3) );
    rule( Qarm3 on Midplane );
    rule( limit_function(Base_translation) );
}
```

The `var` statement ensures that the angles for the rotations for the arms should be modified along with the translation of the base.

Within the body of the `Manipulatorsolve` function, the call to `Redefine_midplane` ensures that the plane which bisects the angle between two gusset faces is re-evaluated.

Three constraints are applied. These are indicated in figure 14 where the right side shows an enlarged view of the gusset fold. The first constraint is that the tip of the final arm (point `Parm3`) must contact a point (`Push1`) which has been defined near the top of the crease between the gusset faces. The second is that the arm itself must lie in the bisecting plane. More specifically, a point (`Qarm3`) on the final arm must lie in the bisecting plane (`Midplane`). These two constraints ensure that the manipulator arm lies between the gusset faces and so does not penetrate the carton. The third constraint controls the motion of the base. The function `limit_function` becomes non-zero when the base becomes near to its allowable limit and increases as it encroaches more closely.

Stepping through the simulation, initially without the third constraint, resolving the other two constraints at each stage, allows the required guidance motion to be found. Figure 15 shows stages in the erection.

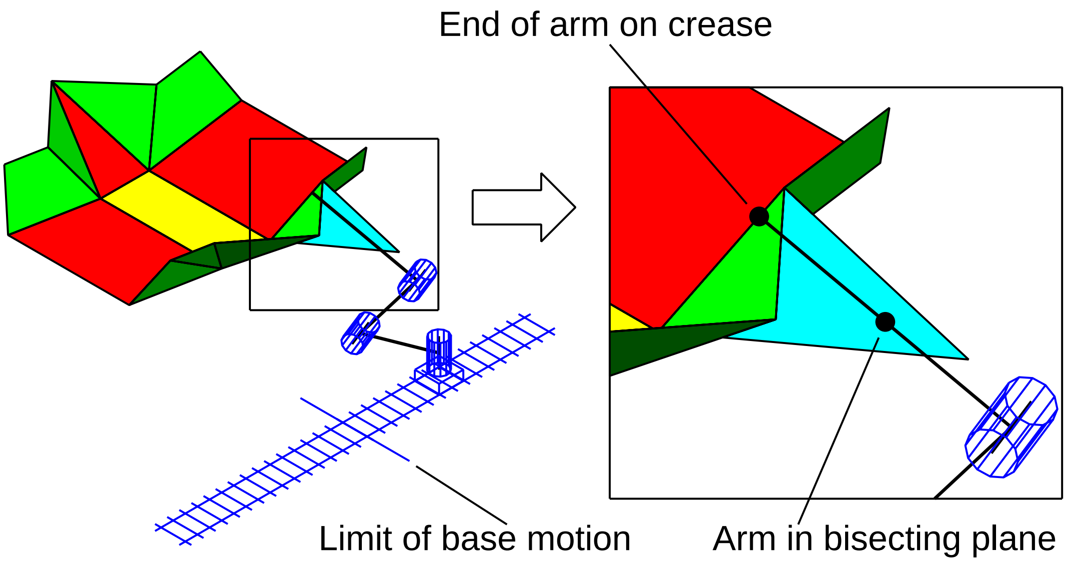

Figure 14: Constraints relating carton and manipulator motion

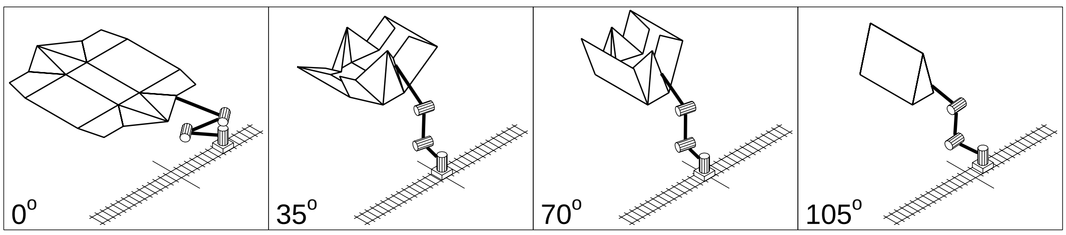

Figure 15: Guidance using a manipulator with four degrees of freedom

This simulation reveals that the base of the manipulator passes beyond the mid-point of the track. This means that it collides with the companion manipulator guiding the gusset on the other side of the triangular face. So the third constraint is added so that the base does not pass the mid-point. The improved motion is shown in figure 16.

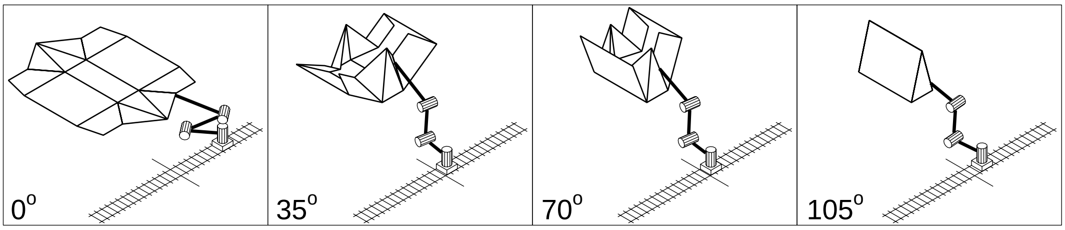

Figure 16: Guidance using a manipulator with four degrees of freedom and constraint on motion of base

Since the model of the manipulator is in terms of the joint angles and the position of the base, it is a simple matter to output these as the simulation runs. These can then be used to control the physical manipulator guiders. Figure 17 shows graphs of the values plotted against the angle of the rectangular side. There are large initial variations in the graphs due to the initial rise in the graph in figure 10. This creates a significant part of the erection process. After this the graphs settle to more constant values as the latter stages of erection take place.

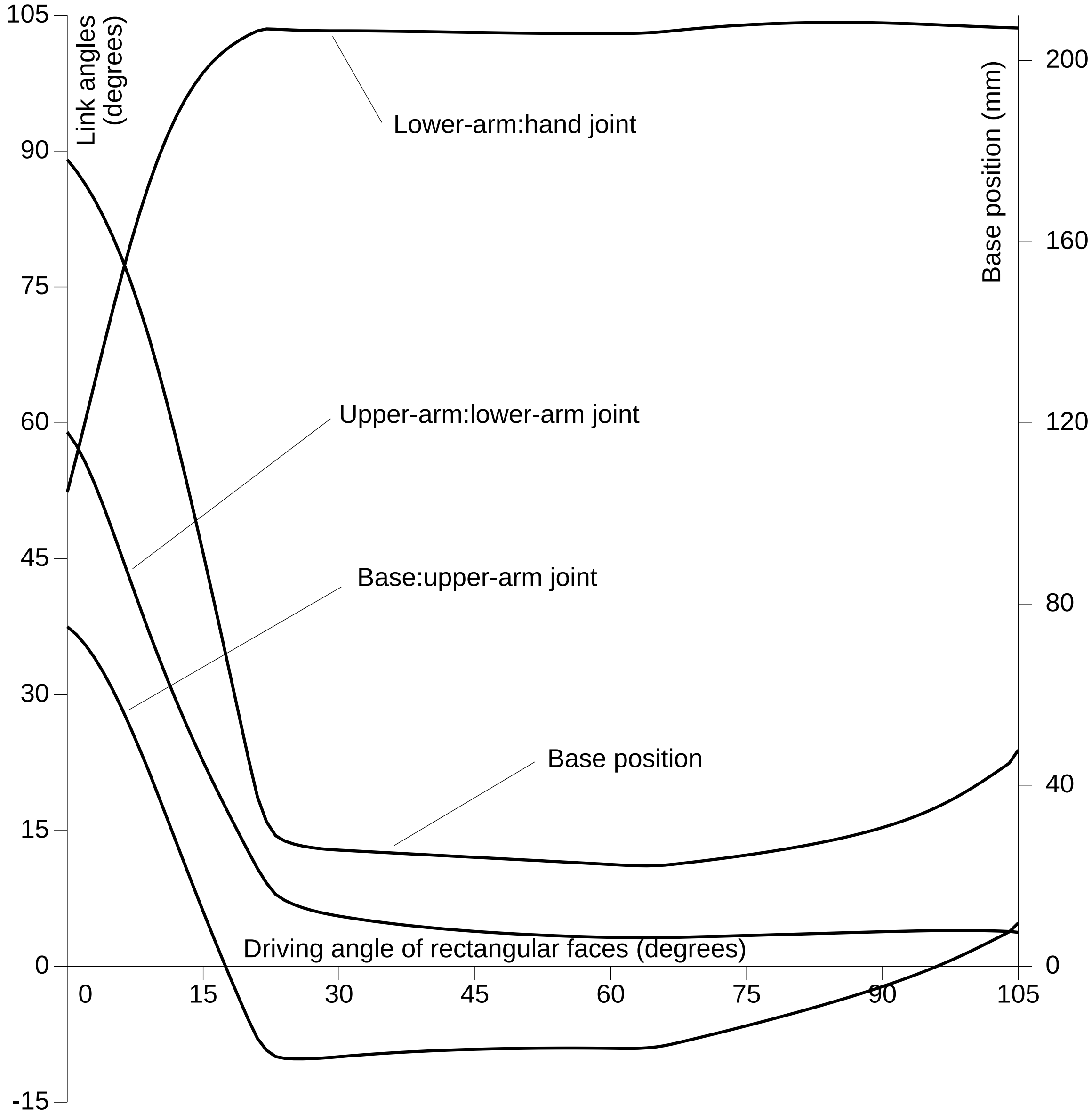

Figure 17: Graphs of manipulator configuration for simulation with four degrees of freedom

6. Conclusions

In the early (conceptual) stages of design associated with the carton and/or with the mechanisms needed to erect it, the precise form of the design is far from being known. This means that conventional mathematical and computer models cannot be set up. What are available are some of the constraints which limit what design decisions can be made. As the design proceeds more constraints become apparent and the understanding of the design task increases. In this paper it has been seen that dealing with the constraints within a suitable software environment can facilitate the initial design investigation. Here the software tool and the human designer work together in creating possible design approaches (which, when complete, can be modelled by conventional means).

In particular, the following points have been demonstrated.

- The relations of the geometry of the underlying carton net can be represented in terms of constraints. This means that a a (simple) initial simulation can be created in a straightforward manner.
- The initial constraint-based model can be extended (by the inclusion of additional constraints) in order to further explore the erection process

and to investigate possible improvements.

- Such extension in the model can include the constraints required to describe the mechanical system used to drive and control the erection process. This allows the carton design and the mechanisms used to erect to be investigated together.

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Figure captions

Figure 1 Loop-free carton net and face graph based on example in [8]

Figure 2 Face graph of carton net

Figure 3 Carton faces forming a dyad

Figure 4 User interface for constraint modelling environment showing the carton erection model

Figure 5 Initial net for carton

Figure 6 Stages in carton erection

Figure 7 Use of solid intersection to identify interference of carton faces

Figure 8 Interference between carton faces

Figure 9 Area of partially closed carton available for fill under different erection regimes

Figure 10 Relation between driving angles for rectangular and triangular faces for improved central area of carton filling: left - initial relation; right - with smoothing to avoid induced jerk

Figure 11 Simulation of erection when using guidance with one degree of freedom

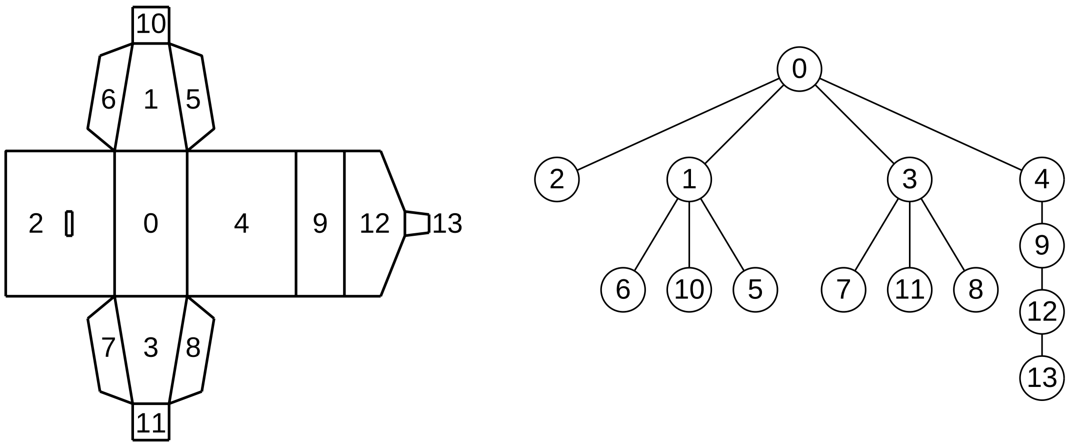
Figure 12 Relation between driving angles when using guidance with one degree of freedom

Figure 13 Manipulator and initial carton

Figure 14 Constraints relating carton and manipulator motion

Figure 15 Guidance using a manipulator with four degrees of freedom

Figure 16 Guidance using a manipulator with four degrees of freedom and constraint on motion of base



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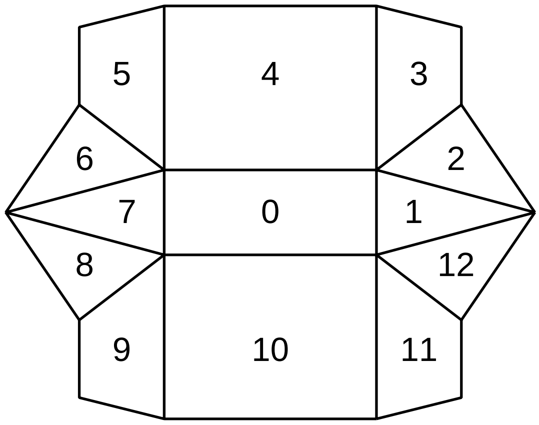

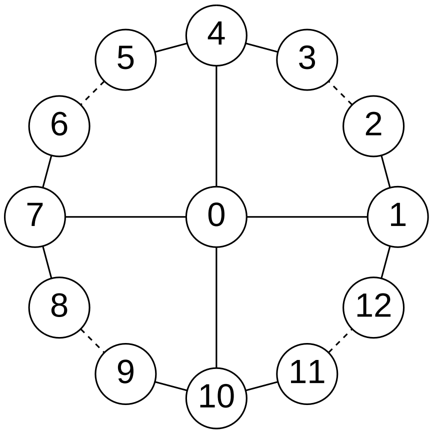

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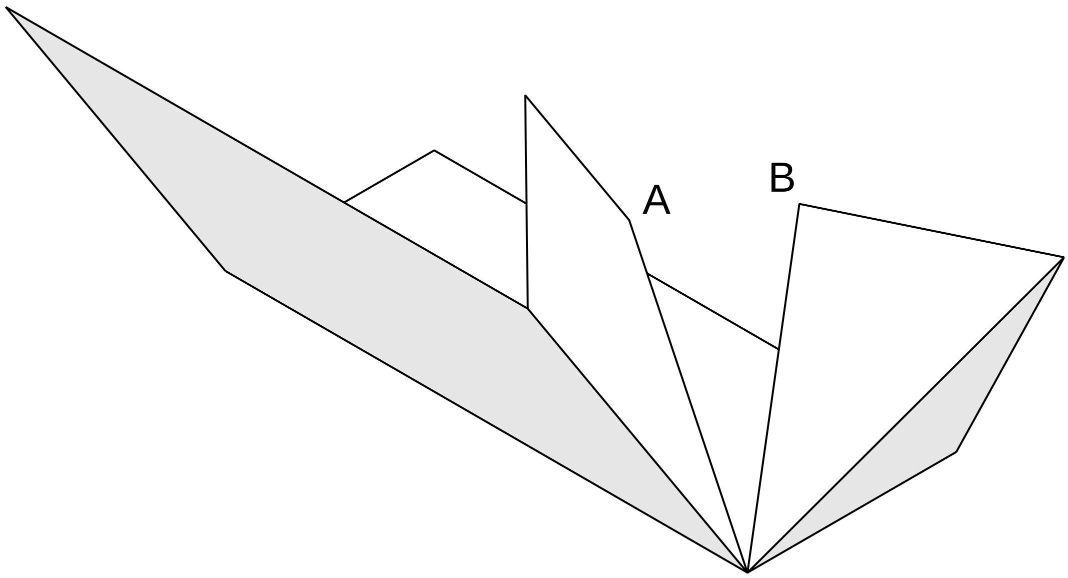

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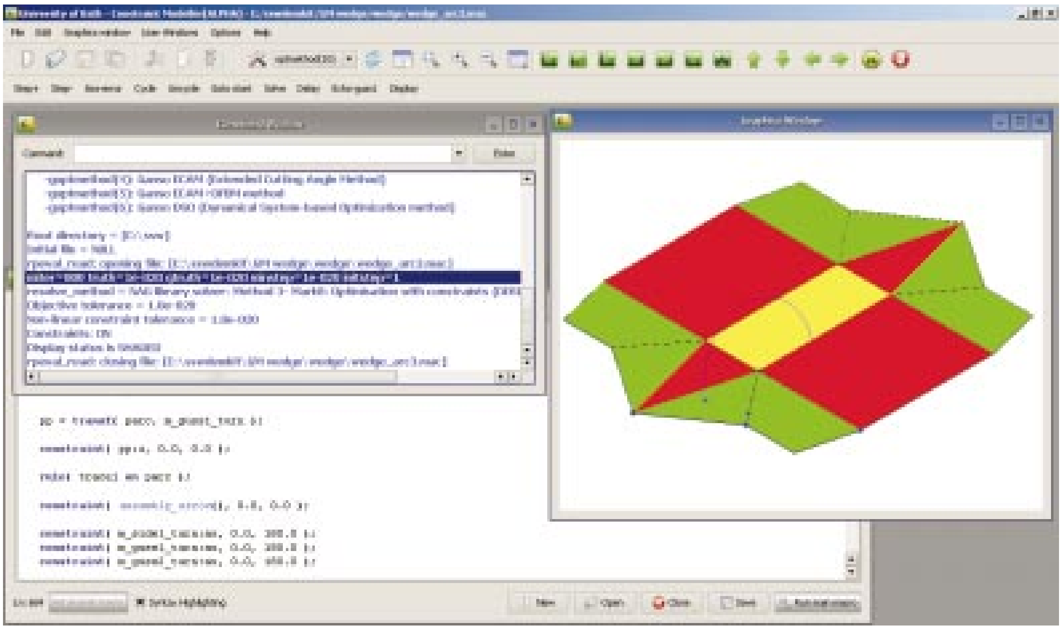

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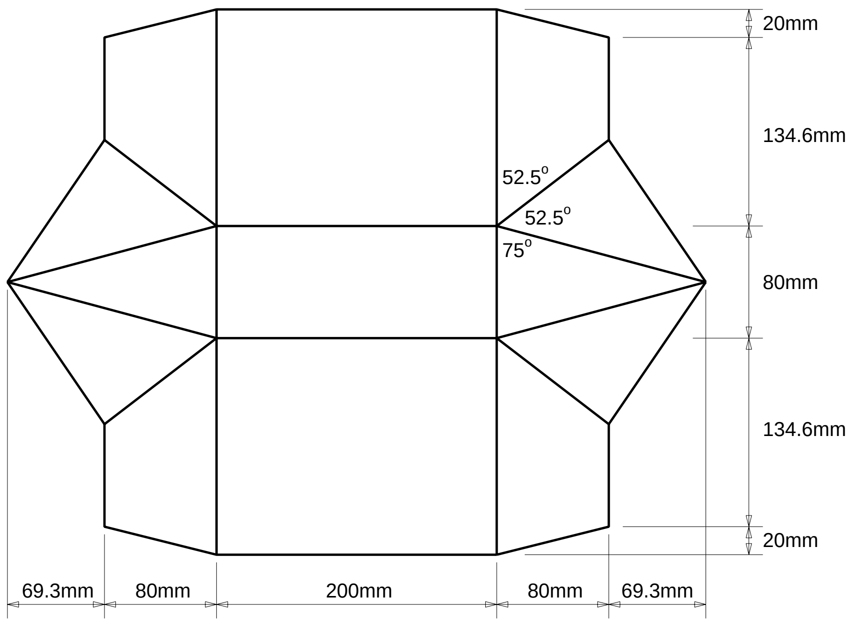

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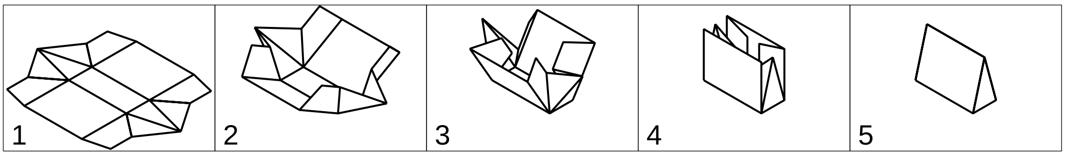
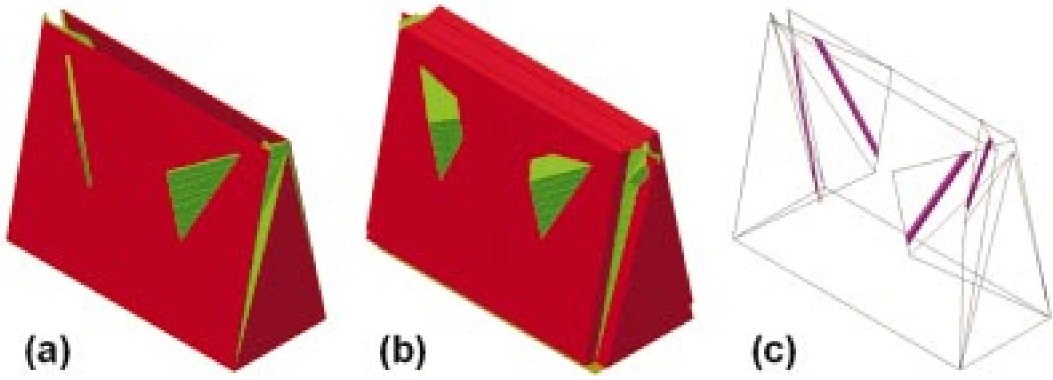


Figure 6: Stages in carton erection



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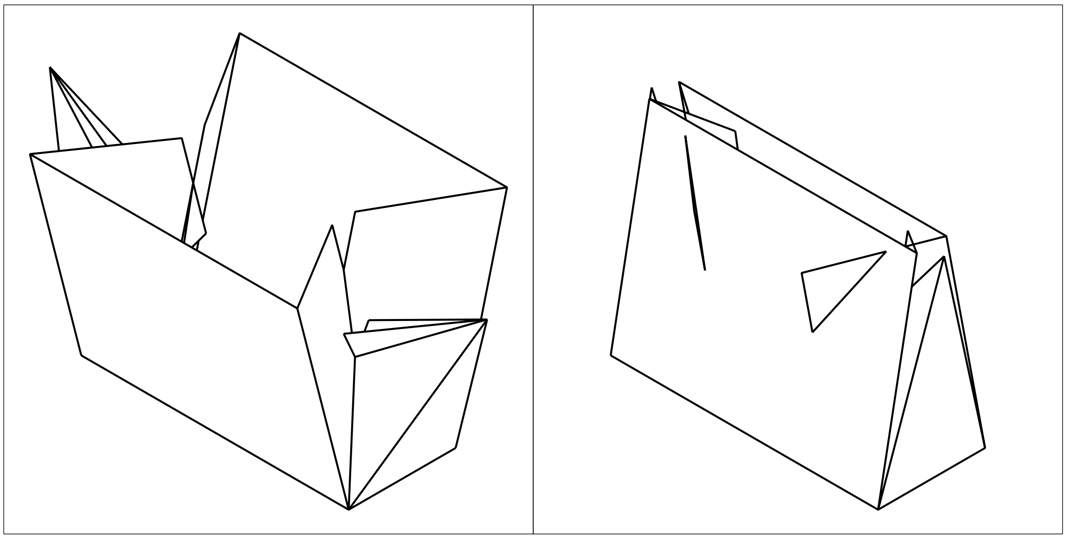
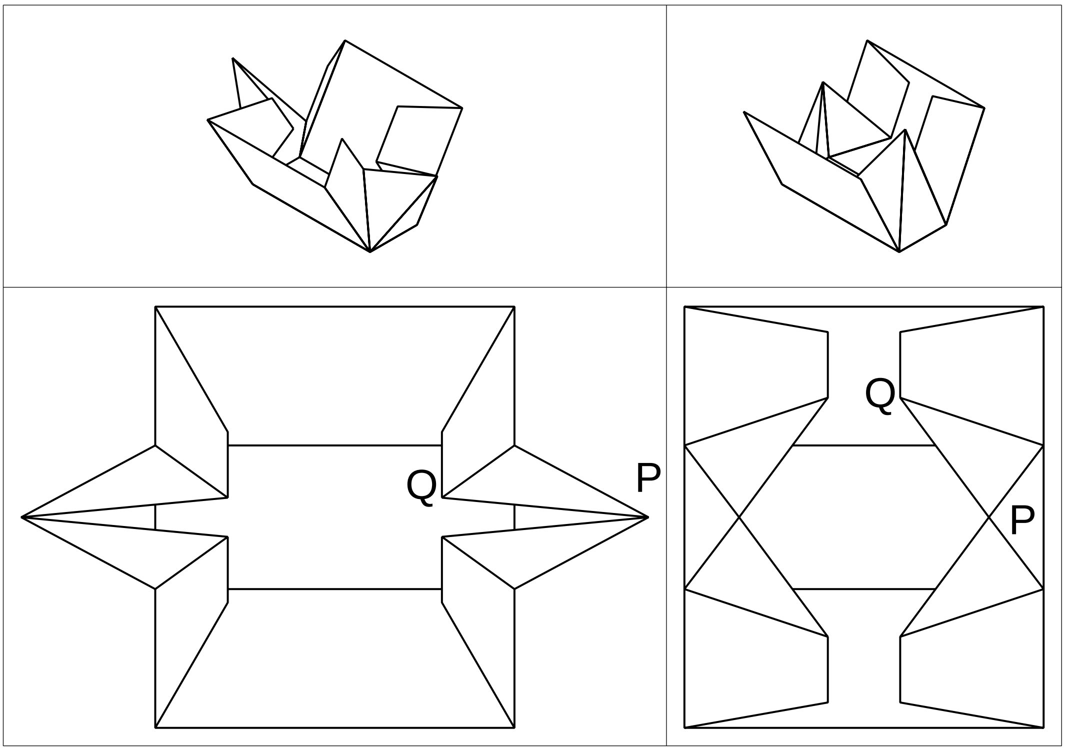


Figure 8: Interference between carton faces



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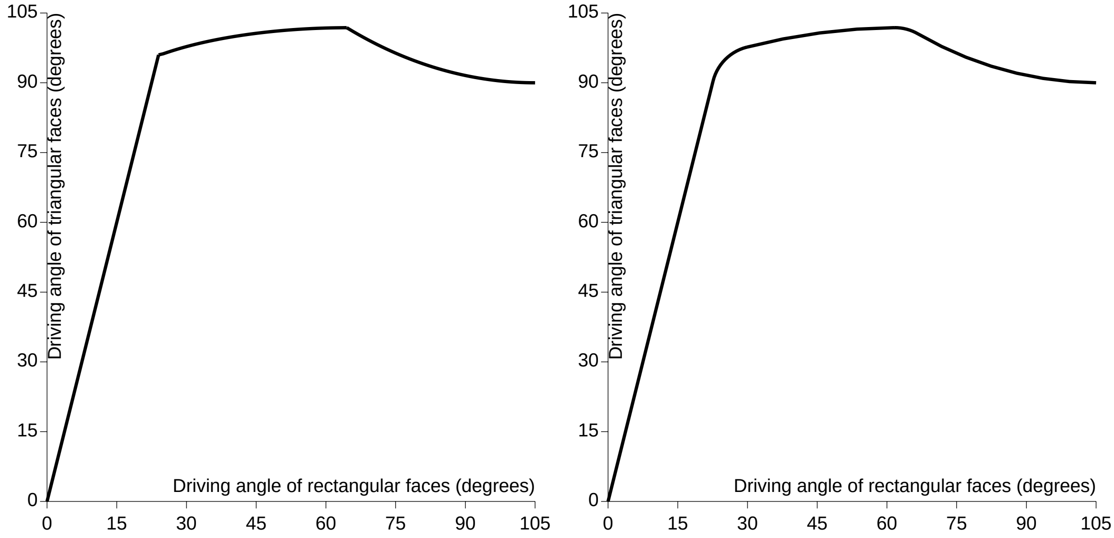


Figure 10: Relation between driving angles for rectangular and triangular faces for improved central area of carton filling: left - initial relation; right - with smoothing to avoid induced jerk

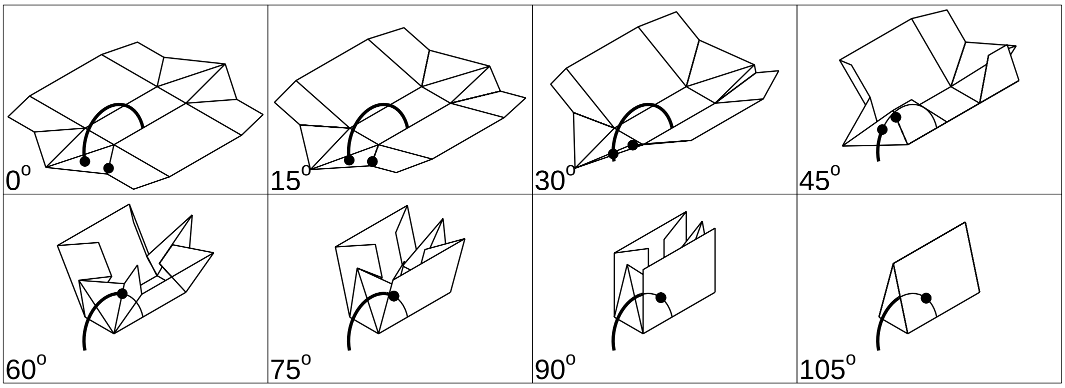


Figure 11: Simulation of erection when using guidance with one degree of freedom

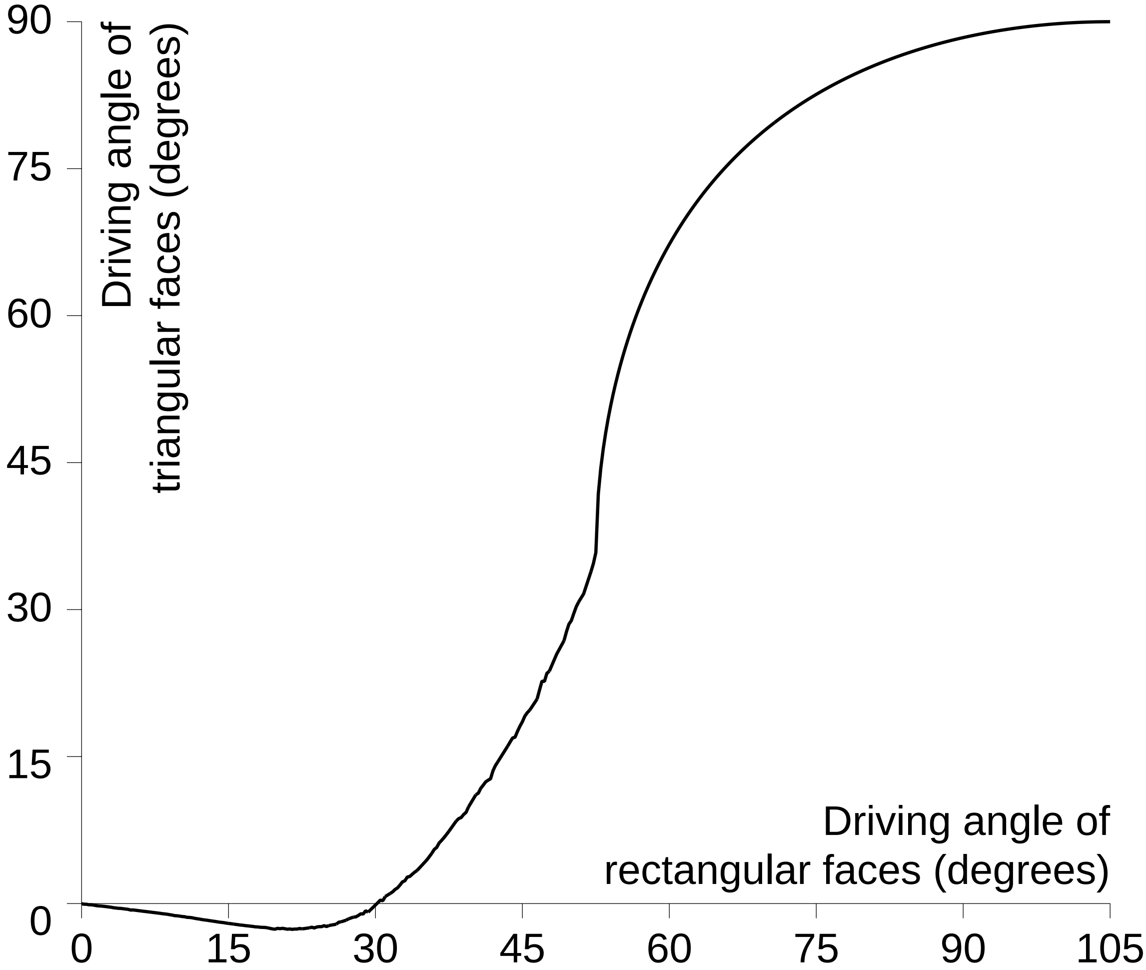
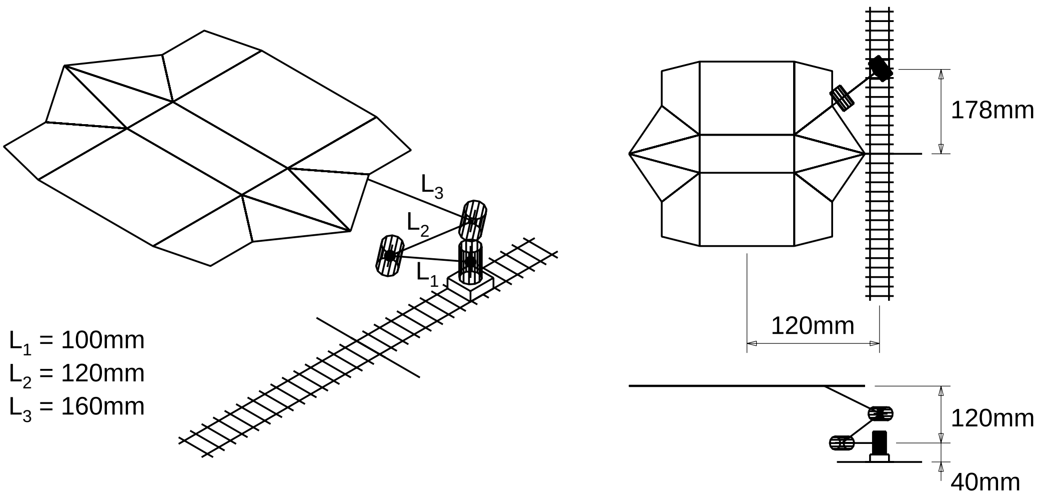


Figure 12: Relation between driving angles when using guidance with one degree of freedom



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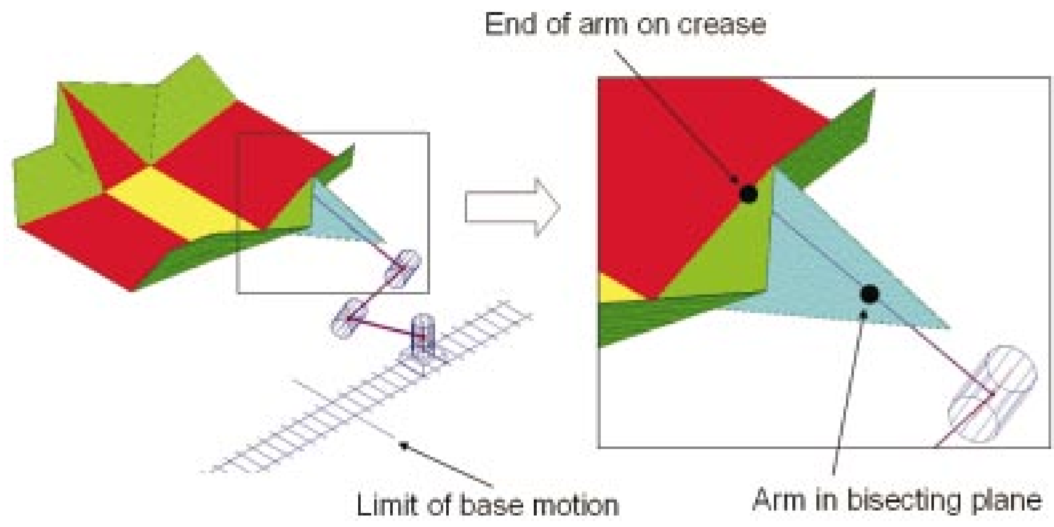

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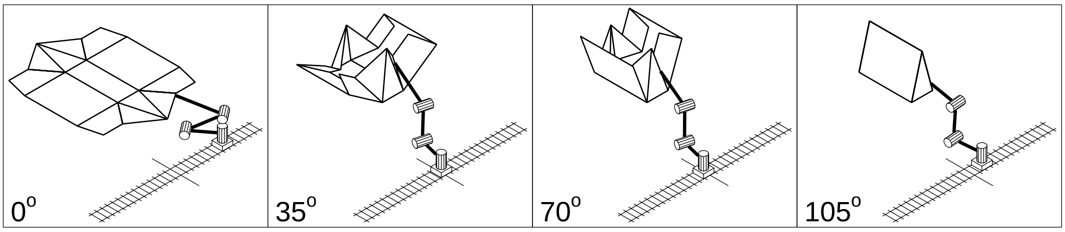
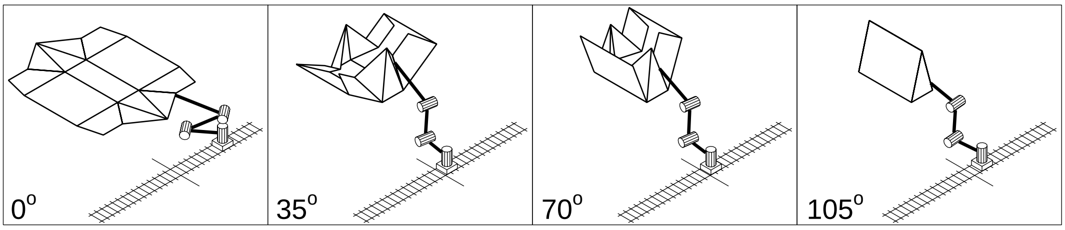


Figure 15: Guidance using a manipulator with four degrees of freedom



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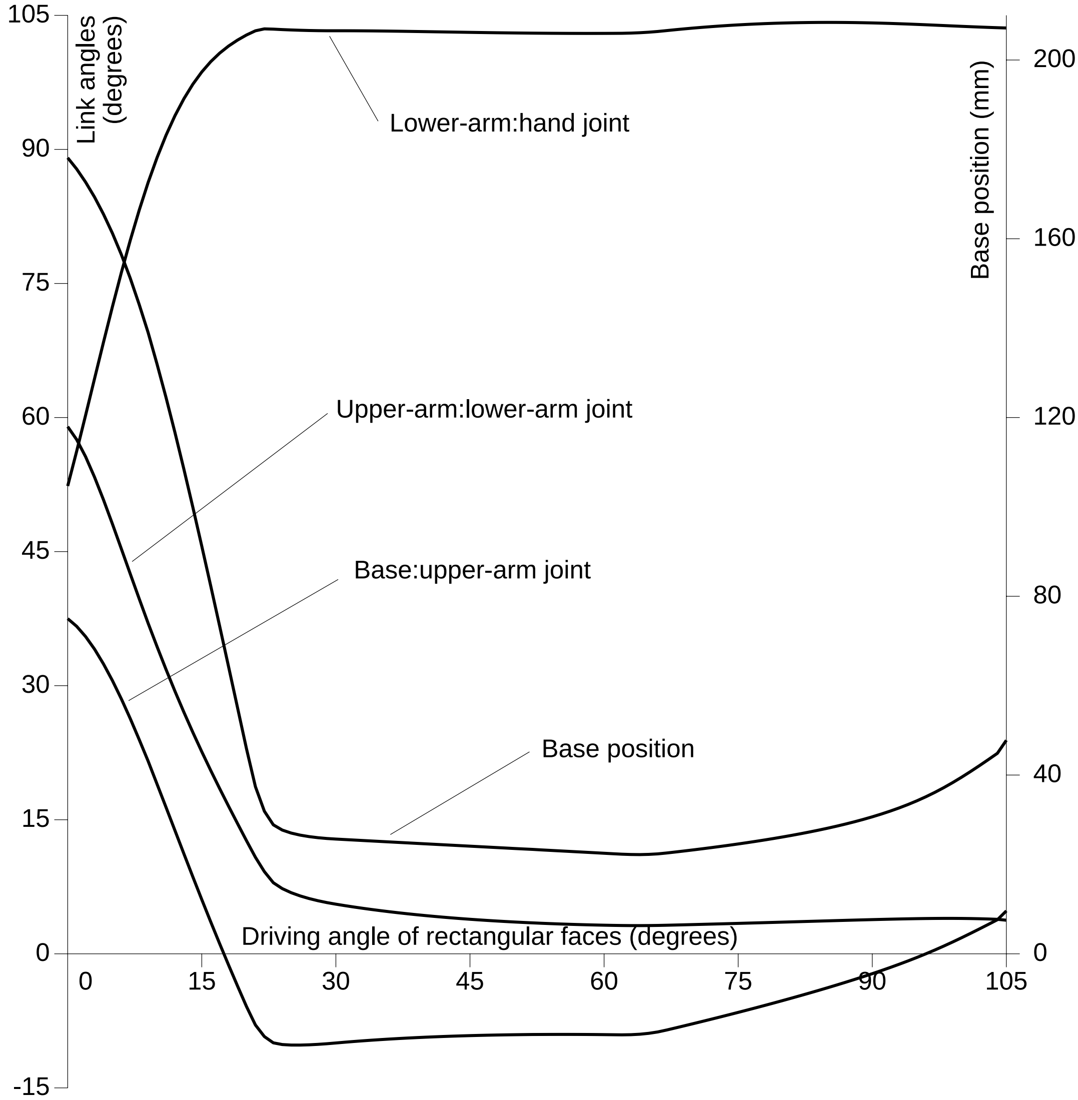

Figure 17: Graphs of manipulator configuration for simulation with four degrees of freedom